New ways in analytical calculation of laminates and composite structures

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Abstract

For the analytical calculation of laminates and composite structures a summarized state of the art calculation recipe as well as a modern design software and material database have been missing so far. This motivated the authors to gather the latest calculation methods and compile them into a software system. The calculation is based on the Classical Laminate Theory (CLT). Main topics are the failure criteria, the consideration of residual stresses and the calculation of the failure process after first ply failure. The new failure criterion and failure process calculation according to Puck are explained in detail. Puck's Action Plane Criterion distinguishes between the different fracture modes, calculates the fracture plane angle for inter fiber failure and can predict catastrophic wedge effects. Puck's progressive failure analysis is based on a new non-linear ply degradation model, which allows a realistic simulation of the failure process. Experimental tests of Puck's theory show a great conformity with reality. The developed software system combines these new calculation methods with all standard CLT and micromechanics calculations, the calculation of structures, an interface for FEA program as well as a full-featured material database.

1. Introduction

The basics of analytical laminate calculation are widely known and are well documented in the respective literature. However for research and design work on a daily basis the following tools have been missing so far:

- a summarized calculation recipe including all state of the art models and calculation methods
- a modern user-friendly software system which realizes this recipe
- a powerful material database.
Needing these tools in their everyday work, the authors decided to develop their own composite material database and design software. With the support of composite design specialist from different universities and the industry, the latest calculation methods were gathered and integrated into a single software system. This development led to a deeper insight into the nature of computerized analytical laminate calculation.

The Classical Laminate Theory (CLT) is the basis for the calculation of composite laminates. Its basic concepts can be found in the respective literature [e. g. 1, 2]. The CLT allows the calculation of the laminate's properties as well as the stress and strain distribution for each ply in the laminate. Once the stresses in the plies are known the strength of the laminate can be calculated by applying a failure criterion to each ply. The strength of the weakest ply is the first ply failure strength of the laminate, which generally is not its ultimate strength. The laminate's ultimate strength can be predicted by progressive failure analysis after the first ply failure.

The needed properties of the laminate's plies can either be measured in tests or evaluated by micromechanics. The data resulting from the laminate calculation can be used to analytically predict the behavior of simple composite structures like plates, tubes or beams.

2. The right choice of the failure criterion

To calculate the strength of a laminate the stress state in each ply has to be submitted to a failure criterion.

The first criteria developed for unidirectional plies were global failure criteria. Global criteria do not distinguish between failure modes. They were formulated as a single mathematically simple equation (the calculation power of modern computers was not available yet) which can be easily adapted to the experimental results. Examples for such criteria are: Hoffmann [3], Tsai-Hill [4] and Tsai-Wu [4]. Until today one of the most commonly used criterion is the Tsai-Wu criterion.

Physical observations led to the development of differentiating criteria, which distinguish between fiber failure (FF) and inter fiber failure (IFF). Different mathematical formulations are used for the physically different phenomena FF and IFF. Because the effects of the two failure modes and the methods to avoid them are completely different, it is vital for the designer to know which failure is occurring. Examples for such criteria are: Simple Puck [5], Modified Puck [6] and Hashin [7].

Based on physical models Puck developed one of the most modern differentiating criterion in order to integrate the numerous experimental observations into one theory [8]. This Puck Action Plane Criterion does not only distinguish between FF and IFF but also between three different modes of IFF (Fig. 1, 2).
Especially the phenomenon of oblique IFF with $s_2 < 0$ (Fig. 2: mode C) motivated Puck to identify the ply plane with the highest effort as "action plane" and to transform the fracture calculations for brittle materials into that plane. Whereas the IFF modes A and B are often tolerable, mode C can lead to a catastrophic failure of the complete composite part. If the angle of the fracture plane exceeds $\pm 30^\circ$ the wedge shape of the crack can damage the neighboring plies in the laminate and thus result in an explosive failure of the whole laminate.

Fig. 3 lists the mathematical equations for the different failure modes. Together these equations define a three-dimensional closed surface, the failure envelop, in the $(s_1, s_2, t_{12})$ stress space. Fig. 4 shows an example. Fig. 5 represents the different IFF modes in a $(s_2, t_{12})$ plane of the $(s_1, s_2, t_{12})$ stress space. The fracture plane angle for mode C can be calculated by the equation shown in Fig. 6. The fracture plane angle for modes A and B is $0^\circ$.

<table>
<thead>
<tr>
<th>failure mode</th>
<th>condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>tensile FF</td>
<td>$\frac{\sigma_1}{X_t} = 1$</td>
</tr>
<tr>
<td>compressive FF</td>
<td>$\frac{</td>
</tr>
<tr>
<td>IFF mode A</td>
<td>$\left( \frac{\tau_{12}}{S} \right)^2 + \left( 1 - \frac{P_{12}^+}{t_{12}} \frac{\nu_1}{\nu_t} \right)^2 = 1$</td>
</tr>
<tr>
<td>IFF mode B</td>
<td>$\frac{1}{S} \left( \sqrt{\frac{2}{s_2} + \left( P_{12}^- \sigma_2 \right)^2} + P_{12}^- \sigma_2 \right) = 1$</td>
</tr>
<tr>
<td>IFF mode C</td>
<td>$\frac{\sigma_2}{X_c} = 1$</td>
</tr>
</tbody>
</table>
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\[
\left[ \left( \frac{2 \sigma_{12}}{2(1+P_{22})S} \right)^2 + \left( \frac{\sigma_2}{Y_c} \right)^2 \right] \frac{Y_c}{(-\sigma_2)} = 1
\]

\(X_t, X_c = \) the ply's tensile and compressive strength parallel to the fiber direction
\(Y_t, Y_c = \) the ply's tensile and compressive strength transverse to the fiber direction
\(S = \) the ply's shear strength transverse and parallel to the fiber direction
\(P_{12}^+, P_{12}^-, P_{22}^+ = \) failure envelop parameter (see Fig. 5)

**Fig. 3:** failure mode conditions of the Puck Action Plane Criterion

**Fig. 4:** 3D model of the Puck Action Plane Criterion failure envelope with constant and variable stress vector

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Fig. 5: IFF modes in a \((s_{2}, t_{12})\) plane of the stress space

\[
\cos \phi_{p} = \sqrt{\frac{1}{2(1 + p_{22}^{-})} \left( \frac{R_{22}^{A}}{S} \right)^{2} \left( \frac{p_{12}}{p_{22}} \right)^{2} + 1} \quad \text{with} \quad R_{22}^{A} = \frac{S}{2p_{12}} \left( \sqrt{1 + 2p_{12}^{-} \frac{p_{22}^{-}}{S}} - 1 \right)
\]

Fig. 6: calculation of the fracture plane angle for mode C

Another important feature of the Puck Action Plane Criterion is the consideration of the interaction between the stresses in fiber direction \((s_{1})\) and transverse to the fiber direction \((s_{2}, t_{12})\). If \(s_{1}\) comes close to the FF limit, the first cracks of single filaments will cause damage in the neighboring matrix. These matrix micro cracks then reduce the plies' IFF limit. This results in the tapering of the failure envelope towards the FF limit (Fig. 4).

But which failure criterion should be used? For many applications global criteria are not detailed enough. Their results can differ decisively from reality. Therefore only differentiating criteria should be used [9]. The Puck Action Plane Criterion is one of the most detailed differentiating criterion. As described in section 5 it is also one of the criteria which corresponds best with experimental results.

3. Hygrothermal stresses and the concept of constant and variable loads

Changes of temperature (operating temperature is different from the curing temperature) and moisture content (matrix material absorbs moisture after curing) lead to thermal or hygroscopic residual stresses inside the laminate [1]. Together with the external mechanical loads these stresses result in the total load, which affects the laminate.

Residual stresses are often ignored. But they must not be ignored. For each ply the combination of the residual and the external stress vector has to be calculated and taken into consideration in the failure criterion.

To be able to answer the question "How far can I increase, under a given constant residual stress vector, the external
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load until failure occurs?" and also the question "How far can I change, under a given constant external load, the operating temperature until failure occurs?" it is necessary to introduce a constant and a variable load (Fig. 7). Since the constant as well as the variable load can be either mechanical, hygrothermal or a combination of both the designer has all calculation possibilities. The total stress vector is then \( \sigma_{\text{total}} = \sigma_{\text{const}} + \sigma_{\text{variable}} \). To calculate the reserve factor for the variable load individually the term \( \sigma_{\text{const}} + \int_{\text{res}} \sigma_{\text{variable}} \) has to be used in the failure criterion equations.

![Fig. 7: constant, variable and total stress vector](image)

point 1: failure point of the vector \( \sigma_{\text{const}} + \int_{\text{res}} \sigma_{\text{variable}} \)
point 2: failure point of the total vector \( \int_{\text{res}} \sigma_{\text{total}} = \int_{\text{res}} \left( \sigma_{\text{const}} + \sigma_{\text{variable}} \right) \)
point 3: failure point of the constant vector \( \int_{\text{res}} \sigma_{\text{constant}} \)
point 4: failure point of the variable vector \( \int_{\text{res}} \sigma_{\text{variable}} \)

4. Degradation models and progressive failure analysis

Together with degradation models the progressive failure analysis can simulate the laminate's failure process after the first ply failure.

After a ply in the laminate reaches its IFF limit, local matrix cracks will appear. These cracks do not lead to the total loss of this ply's strength since neighboring laminate layers bridge them. However there is a reduction in the ply's stiffnesses. This results in a new distribution of the stresses in the laminate. With increasing load beyond the IFF limit more and more local matrix cracks will appear until a certain saturation is reached [8, 9].

In laminate calculation the physical reduction of the stiffnesses is taken into account by degrading the ply i.e. by mathematically decreasing the ply's stiffnesses. In the respective literature only a few degradation models can be found. Mostly the plies are degraded by a constant factor directly after the IFF [2, 4, 10]. This does not reflect the crack building process as described above. Together with his new Action Plane Criterion Puck introduced a new degradation model [8]. The degradation of the ply follows a non-linear function of the effort.

In order to simulate this process on the computer an iterative algorithm is used where the load is increased step by step. After each step the complete laminate as well as the stress distribution has to be recalculated and over stressed plies have to be degraded.

Normally a fiber failure in the laminate is not acceptable and the progressive failure analysis is stopped. However laminates with several fiber orientations can still have load-bearing capacities. Consequently it can be interesting to continue the calculation beyond the first fiber failure. In this case the fiber-failed plies are degraded by setting their stiffnesses to zero. Just their geometrical dimensions and positions in the laminate are kept. The calculation process can be continued until the last ply fail in fiber direction.

5. Experimental results of Puck's theory

Puck's Action Plane Criterion and degradation model show high conformity with experimental results. Tests conducted at the IKV (Institute for Plastics Processing), Technical University Aachen, Germany, were positive [11, 12].

A detailed evaluation and comparison with other failure criteria has been established in the "Worldwide Failure Exercise" by Hinton, Kaddour and Soden [13, 14, 15]. This exercise compared 12 different failure theories on 14 different test cases. An overall assessment concludes that the Puck theory "appears to be one of the best available currently".

In their practical design work the authors used Puck's action plane criterion successfully for many filament-winding projects.

It should also be mentioned that the criterion and degradation model is the new standard in VDI (Association of German Engineers) guideline 2014 [9].

6. Software for the real world

The software developed by the authors combines the above discussed features with all standard micromechanics, CLT and structure calculations. Fig. 8 shows the structure of the program and its database. Besides the calculation capabilities the following features have been integrated.

**Database**

To be able to use the calculation capabilities of the software efficiently a powerful database is needed. This database stores all data of fibers, matrices, plies, laminates, stacking sequences, loads and structures. Features like searching, filtering and sorting allows direct access to the data for immediate calculation. In addition to physical and engineering parameters information like comments and text notes or quality control data can be stored.

**FEA interface**

Laminated beams, tubes and plates belong to the few structures for which an exact elasticity solution can be found. More complex structures have to be analyzed by finite element analysis (FEA). This fact makes it necessary for the software to be able to exchange data with FEA programs. Material data from the database as well as calculation results can be exported to and material, laminate and structural data can be imported from FEA programs.

**Graph engine**

One of the best way to present data is a graph. The software includes a graph engine, which allows the display of any X-Y or polar graph, including any ply's property versus the ply's orientation angle, carpet plots, ply-by-ply stress and strain diagrams and failure process diagrams. An important tool is the display of the 3D failure envelope surface in the $(s_1,s_2,t_{12})$ stress space (Fig. 4).
User interface

The difficulty with the designing of the user interface has been how to present the large number of input and output data to the user in an efficient way. For example more then 1500 values are calculated per ply in the laminate. For a laminate of 100 plies this results in 150,000 values. Not all these values are equally interesting for the user. A way had to be found to divide this data into groups and to display only the currently interesting values. This problem has been solved by using table grids, which can be configured individually by simple mouse click and allow a spreadsheet style use of the software.

Fig. 8: program and database structure

7. What's next?

The authors will continue to observe the research on the field of laminate and composite structure design. New developments will be included in the software. Future ideas for the software include:

- considerations of non-linearities in the stress-strain-relations
- calculation of more complex structures (e.g. thick wall parts, pressure vessels)
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- integration of simple FEA capabilities
- consideration of interlaminar stresses

8. References

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9. BIOGRAPHIES

Axel Seifert (mechanical engineer) and Christian Laval (electrical engineer) worked from 1986 to 1990 in the composite department of the IKV (Institute for Plastics Processing) at the Technical University Aachen, Germany. In
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